3D Numerical Modeling of Flows at the Confluence of the Columbia and Pend d'Oreille Rivers

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Abstract

A high resolution three-dimensional numerical model, COCIRM, was adapted and optimized to predict the potential impact of the Waneta Expansion Project (WEP) on the flow conditions at the confluence of the Columbia and Pend d'Oreille rivers. Flows in this confluence area represent important habitat features for white sturgeon, including the well-developed low-speed Waneta Eddy for sturgeon rearing and feeding, and the jet-like high-speed Pend d'Oreille River outflow for sturgeon spawning. Extensive model calibrations and validations were carried out to render the model a useful and reliable tool for the WEP environmental impact assessment. The model results are in good agreement with extensive in situ observations in terms of the patterns of the Waneta Eddy, Pend d'Oreille outflow and its associated standing waves as well as water temperatures.

Introduction

Columbia Power Corporation is proposing to develop the Waneta Expansion Project that includes a new powerhouse located on the right bank of the Pend d'Oreille River downstream of the existing Waneta Dam (Figure 1). The expansion project may affect the flow and circulation patterns in the area of the confluence of the Columbia and Pend d'Oreille rivers. Previous studies revealed that this confluence area has some significant morphological and circulation features, which are important for white sturgeon such as the deepwater low-speed Waneta Eddy for sturgeon rearing and feeding and the jet-like, high-speed Pend d'Oreille River outflow for sturgeon spawning (Hildebrand and Fissel, 1997; Hildebrand, 2001).

The confluence of the Columbia and Pend d'Oreille rivers is located about 500 m north of the Canada and USA border (Figure 1), where flows from the upstream Columbia River and the Pend d'Oreille River join together before passing into the United States. The center of the confluence is a region of a large embayment with a water depth up to 18 m or more, much deeper than the surrounding areas. The flows in this area appear comparatively weak, typically less than 0.5 m/s, and usually rotate in counter-clockwise direction, known as the Waneta Eddy. On the northern side of the Waneta Eddy, the main channel of the Columbia River features large flows through typical water depths of 2 to 6 m relative to chart datum at 394 m above mean sea level (MSL). Just upstream of the confluence of the two rivers, a large gravel bar extends out from the eastern shore which confines the main flows of the Columbia River to a comparatively narrow channel along the western shore. Immediately to the south of the Eddy is the strong jet-like outflow from the Pend d'Oreille River into the

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Columbia River. The flows under the road and rail bridges, where the Pend d'Oreille waters enter the Columbia River, are normally turbulent due to the shallow and narrow Pend d'Oreille passageway. This discharge zone is characterized by large standing waves indicative of a supercritical flow regime. The highly turbulent, standing wave area extends downstream as far as the southeast corner of the deep portion of the Waneta Eddy.

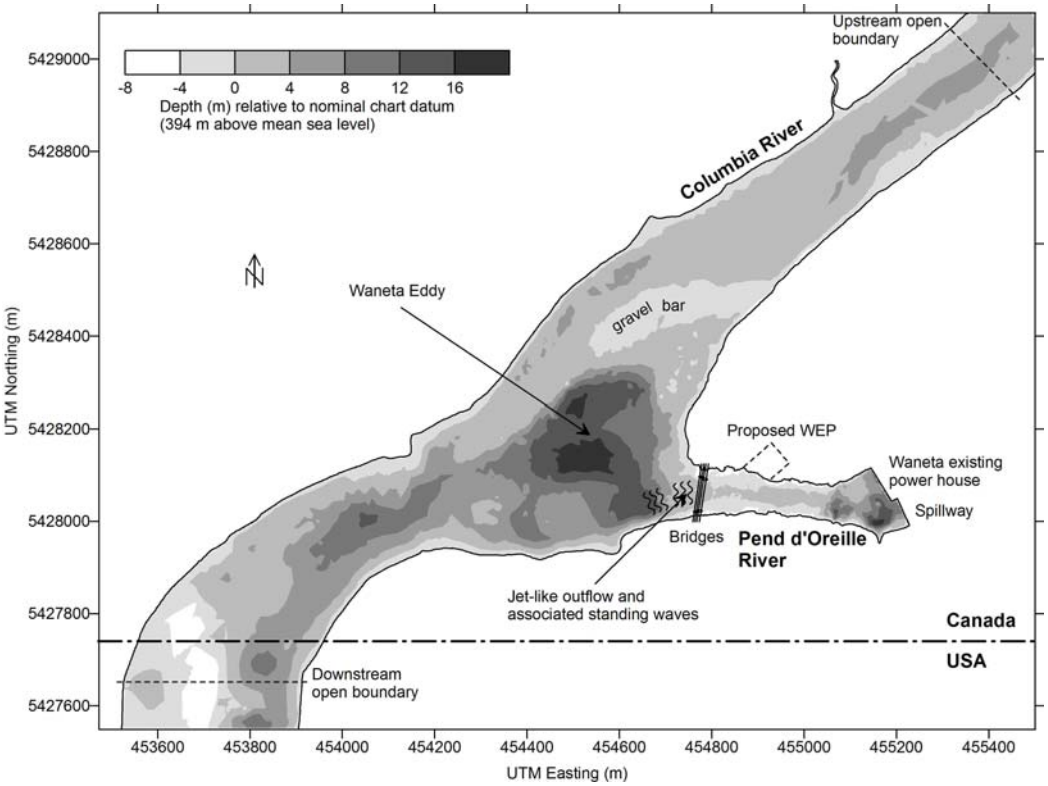

Figure 1. Study area showing geometry, model domain and boundaries.

The implement of the 3D COCIRM has been aimed at providing a very useful and reliable tool for application to various environmental impact assessment issues addressed as part of the regulatory approval process for the proposed WEP. This paper describes the modeling approach, calibration, and validation results in terms of comparisons with in situ observations of flow, water level and water temperature, followed by quantitative evaluation of the model performance. The potential impact of the WEP on the flow in the confluence area was examined using extensive model predictions with numerous flow combinations from the Columbia River, the existing Waneta Dam and the WEP, and the predicted results for various scenarios will be presented in another paper.

Model Input Data and Boundary Conditions

Model Area and Bathymetry

The model domain is centered on the confluence area, extending approximately 1260 m upstream from the Eddy and 1050 m downstream to an area just south of the Canada and US border. The upstream boundary of the model on the Pend d'Oreille River is the Waneta Dam, situated about 450 m upstream of the

entrance of the Pend d'Oreille River into the Columbia River. The model runs on a grid with 3 m by 3 m horizontal grid cell size and 10 equally-spaced sigma layers.

Bathymetry data from surveys before 2001 and during 2003 – 2004 were carefully reviewed to rule out any inconsistencies and checked for sufficient resolution and important morphological features, especially the gravel bar and outflow regions. The final bathymetry data used in the model have the resolution of 3 to 6 m or better in most areas, such as the Waneta Eddy, gravel bar, and Pend d'Oreille River and its outflow regions. The bottom features in these regions are resolved appropriately through interpolating bathymetry data onto the model grid.

Boundary Conditions

The upstream boundary of the Pend d'Oreille River in the model is the existing Waneta Dam. The representation of the existing Waneta tailrace and spillways in the model was based on as-built drawings in a realistic manner. The draft tubes are located at near-bottom 1 through 5 sigma-layers, and the resulting currents were oriented in the same direction as the tailrace. The flow from the spillway has relatively large momentum, due to the acceleration of the water as it falls down the spillway just before it enters the water column in an almost vertical direction. The part of this kinetic energy is dissipated through turbulence, and the remainder is transferred into horizontal flow. The total discharge from a particular spillway was input to the model in accordance with the conservation of mass principle, into the mesh grids adjacent to the spillways. The distribution of momentum is adjustable for each sigma layer. In the model runs, the momentum of the spilled water was used to derive the horizontal flow at the boundary grids (Jiang and Fissel, 2002).

The flows at the upper model boundary, on the Columbia River upstream of the confluence, were specified for each of the calibration and validation runs. Field measurements of river flow in the vicinity of the upper Columbia River model boundary were made in support of the present study. A numerical algorithm was developed to partition the total Columbia River discharges into the flow field at this boundary (Jiang and Fissel, 2002).

The boundary conditions for the downstream portion of the Columbia River were specified through a modified form of Sommerfeld radiation approach (Orlanski, 1976) as follows.

$$\frac{\partial \vec{n} \cdot \vec{V}}{\partial t} + C_G \frac{\partial \vec{n} \cdot \vec{V}}{\partial n} = 0 \qquad (1)$$

where t is the time, $\vec{n}$ is the unit vector normal to the open boundary, $\vec{V}$ is the velocity vector at the open boundary, and C_G is the propagation speed of disturbances. The initial water elevation at this open boundary was derived from stage discharges at the Canada/USA border International gauging station operated by the US Geological Survey and the Water Survey of Canada. The particular initial water level chosen for each model run was derived as the sum of the Upper Columbia River and Pend d'Oreille River discharges specified and converted into a water level at the Canada and USA border from the stage discharge curve.

Model Stability

The stable time step, *dt*, is determined by a stability constraint resulting from the semi-implicit, finite-difference numerical scheme (Casulli and Cheng, 1992) applied in COCIRM. In the present study, *dt* was taken as 0.67 sec. Model run starts with the initial conditions of quiescent water and flat surface. The constant water elevation was set to the initial water level at the downstream open boundary, derived from the stage discharge curve at the international border. In the confluence area, especially in the Pend d'Oreille River outflow and the strong shear zone between the outflow and the Waneta Eddy, the model results can exhibit periodic fluctuations. The model results became stable if maximum velocity fluctuations were less than 0.05 m/s. This process takes 1 to 2 hrs of real time and consumes computer time of about 12 hrs on a Pentium IV computer with 3 GHz processor and 4 GB physical memory.

Model Calibration and Validation

In the confluence area, the flow patterns and the outflow standing waves, etc., appear to be dynamic and vary considerably with different flow combinations of the Columbia and Pend d'Oreille rivers. To evaluate the model performance and validate it as a reliable tool for the environmental assessment, the flow combinations of the two rivers in all model calibration and validation cases must span a substantial range to represent various flow features for which the in situ observed current data are available. Since 1994, extensive field measurements of currents in this region, mostly in the confluence area, have been carried out by ASL Environmental Sciences Inc. and R.L. & L. Environmental Services Ltd., using ship-borne ADCP (Birch, 1994; Birch, 1996; Hildebrand and Fissel, 1997; Birch and Boubnov, 2001; Birch and English, 2001). These data allow extensive amounts of opportunity for model calibration and validation in this study. In total, two calibration cases (C1 and C2) and seven validation cases (V1 – V7) were selected and carried out. These cases are believed to be sufficient for the model validation requirements (Table 1).

Table 1. Summary of model calibration and validation cases.

Model case		Discharge (m^3/s)				Temperature (°)		observation taken
		Upper Columbia River	Pend d'Oreille River			Upper Columbia River	Pend d'Oreille River	
			Exsiting Power house	Spillway	Total			
Cali.	C1	1,812	229		229	18.8	22.4	Aug. 31, 94
	C2	2,300	725	209	934	2.5	1.5	Feb. 08, 96
Validation	V1	1,982	510		510	18.8	22.2	Aug. 30, 94
	V2	951	147		147			Jul. 15, 01
	V3	1,104	227		227			Oct. 20, 01
	V4	1,104	510		510			Oct. 20, 01
	V5	951	283		283			Jul. 15, 01
	V6	2,550	34		34			Oct. 06, 96
	V7	2,039	720	359	1,079	10.0	13.0	May 18, 94

The model was initially tested and operated in calibration runs. Various physical parameters, mainly bottom drag coefficient and horizontal and vertical eddy diffusivity coefficients, were repetitively adjusted to achieve optimal agreement with the observations. The vertical diffusivity for the model, as derived from the second order turbulence closure model (Mellor and Yamada, 1982), was found to be robust. Some adjustments of the horizontal diffusivity were made through the user-specified calibration parameter in Smagorinsky's formula (Smagorinsky, 1963). The bottom drag coefficients were the most important parameter for the purpose of model calibration, as discussed below.

Once reasonable agreement is attained for each of the calibration cases; the model was next operated in validations runs using the previously optimized physical parameters and compared with different observation data sets. The agreement between the model outputs and the observations is used to assess the capabilities of the model. If the comparisons do not meet the model requirements, or indicate that significant further improvements are needed, the calibration processes can be repeated to improve the model performance. The final choice of the bottom effective roughness heights, z_0, is 0.0075 m in the Columbia River upstream of the gravel bar and in the Pend d'Oreille River upstream of the bridges, and 0.0025 m in the remaining areas. Underwater photographs show that bottom materials are mostly rocks and gravels in the upper Columbia and Pend d'Oreille rivers, while mostly sands in the confluence area, especially the deeper portion. Therefore, these optimized z_0 values are considered physically reasonable. The horizontal diffusion coefficient C_A is evaluated using Smagorinsky's formula (Smagorinsky, 1963) and is equal to 0.15.

Flows

The model results of flows were compared with ADCP data at different vertical levels. As shown in **Error! Reference source not found.**, the overall agreements between model and observations in terms of general flow patterns in the confluence area are good. Noteworthy discrepancies between modeled and observed flows mainly appear in the strong shear zone between the Waneta Eddy and the Pend d'Oreille outflow. Detailed data analyses show that the flows in this strong shear zones are highly turbulent and unstable causing considerable uncertainty in both model and observations (Fissel and Jiang, 2002). To provide overall assessment of model performance, further comparisons between modeled and observed flows were conducted using the statistical model validation procedures of Murphy and Winkler (1987). The analyzed results are presented in the next section of "Validation Statistics".

Model results and observations indicate that the circulation patterns in the confluence area vary dynamically with discharge levels from the two major rivers (Figure 1). A local circulation around a geometric feature is mainly driven by flow shears. This is also the case at the confluence of the Columbia and Pend d'Oreille rivers. The confluence flow patterns of all calibration and validation cases are summarized as schematic diagrams in Figure 3 (Jiang and Fissel, 2002). In general, the deep portion of the confluence area is occupied by the well-developed low-speed Waneta Eddy, with the flow speed typically less than 0.5 m/s. At the western and northern sides of the Waneta Eddy, the Columbia River main flow steers around, in

part, under the effect of the gravel bar, with a flow speed usually greater than 1 to 3 m/s. The stronger Pend d'Oreille River outflow is located at the southern side of the Eddy. At the core of the outflow, flow speeds are usually greater than 2 to 3 m/s, with maximum value up to 5 – 10 m/s.

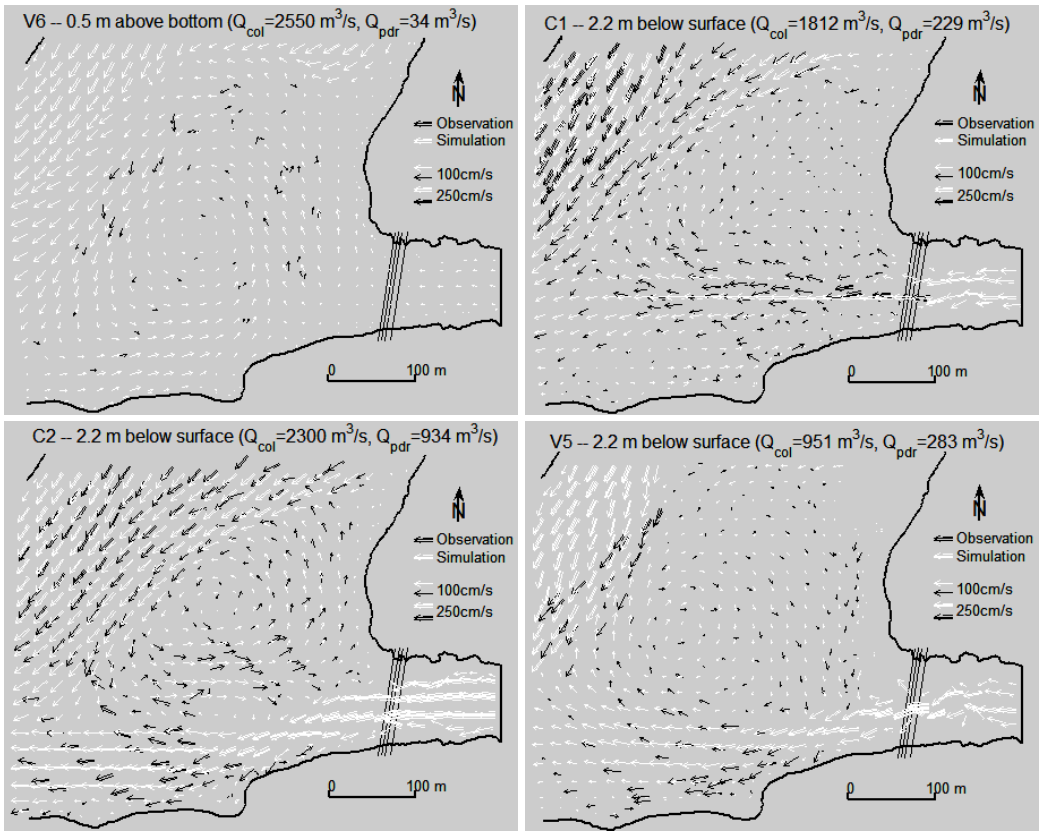

Figure 2. Model results of flows for V6, C1, C2 and V5 in the confluence area, with comparisons to observations.

The confluence circulation patterns vary dynamically with the variations of discharges from upper Columbia River and Pend d'Oreille River. At extremely low Pend d'Oreille River flow, such as the "speed-no-load flow" of 34 m^3/s in the validation case V6, the entire embayment area from the gravel bar to the southern shore is occupied by a large counter-clockwise eddy (labeled as C in Figure 3), driven by the shears of the main Columbia River flow. The weak Pend d'Oreille River flow joins the eddy along the eastern bank. As the Pend d'Oreille River discharge increases, the outflow gains more and more momentum to break through the big eddy. In these cases, the actual circulation patterns are dependent not only on the Pend d'Oreille River flow, Q_{pdr}, but also on the Columbia River discharge level, Q_{col}. For moderate to high levels of Q_{col} (≥ 1,300 m^3/s), the outflow gradually shifts southward towards to southern shore as the Q_{pdr} increases. At the same time, the clockwise eddy (labeled as P in Figure 3), which is driven by the shear of the Pend d'Oreille outflow, gradually shrinks and the counter-clockwise eddy C becomes dominant. During this process, a third eddy appears near the southern shore, namely south shore eddy (labeled as S in Figure 3), driven by the shears from both Columbia main flow and the Pend d'Oreille outflow, and disappears at high level of Q_{pdr}. For the low flow level

(Q_{col} < 1,300 m^3/s), the gravel bar acts as a weir or dam with little or no water passing over it. As a result, the deep water area is dominated by the clockwise eddy P if the Pend d'Oreille River discharges are at low to modereate levels (100 – 500 m^3/s). At the high flow level (Q_{pdr} > 500 m^3/s), the water elevation at the confluence increases and the gravel bar is submerged with a considerable amount of water passing over it. Consequently, the counter-clockwise eddy C becomes dominant again.

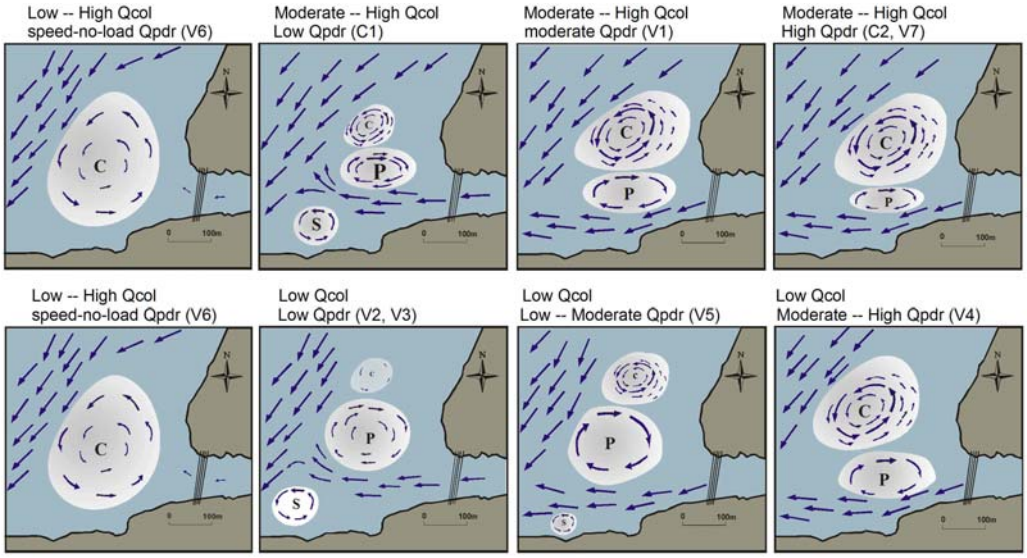

Figure 3. Schematic diagram of the circulation patterns in the confluence area.

Standing Waves

Depending on the discharge level from the Waneta Dam, the outflow speed in the shallow and narrow zone immediately beneath the bridges could be up to 5 – 10 m/s with a water depth typically less than 5 m. This part of the outflow is apparently in a supercritical regime with a Froude number greater than 1.0 – 1.5. As a result, significant standing waves occur downstream of the bridges. The modeled water surface appears to be rough in the area bounded by around 120 m both upstream and downstream of the bridges (Figure 4). Immediately upstream of the bridges, the water elevation drops about 1 to 4 m over a very short distance of about 10 to 30 m. This abrupt water drop appears to be induced by sudden flow cross-sectional area expansion along with supercritical flow conditions (Figure 5). Thereafter, four distinct waves develop within 120 m downstream of the bridges. Dependent on the strength of the outflow, these standing waves have wave heights ranging from 0.2 m to 1.5 m and wavelengths ranging from 20 m to 40 m. These standing waves are highly three-dimensional with the water surface rougher at the center of the outflow while relatively smooth near the shore. There are some small wave features riding on these large standing waves. These modeled standing wave features appear to be in reasonable agreement with field visual and photographic observations (Fissel and Jiang, 2002; Jiang and Fissel, 2002).

No actual water elevation profiles were measured in the supercritical, high turbulent outflow area until early 2007 when a water elevation survey was conducted along the river bank. This survey provides good data to examine the model capability

of simulating standing waves in the supercritical outflow region. Figure 5 shows the comparisons between modeled and observed water elevations for three flow conditions. The model results are in good agreement with the observations in terms of the shapes of the water surface profiles and the big water drop under the bridges. Some discrepancies between the model results and data exist. The model water elevations are lower than the observations by around 0.15 m downstream of the bridges and by around 0.3 m upstream of the bridges. These discrepancies are mainly caused by insufficient/inaccurate bathymetry input. It is also suggested that further refinement of the model parameters, especially bottom drag coefficient, is necessary to further improve the model performance.

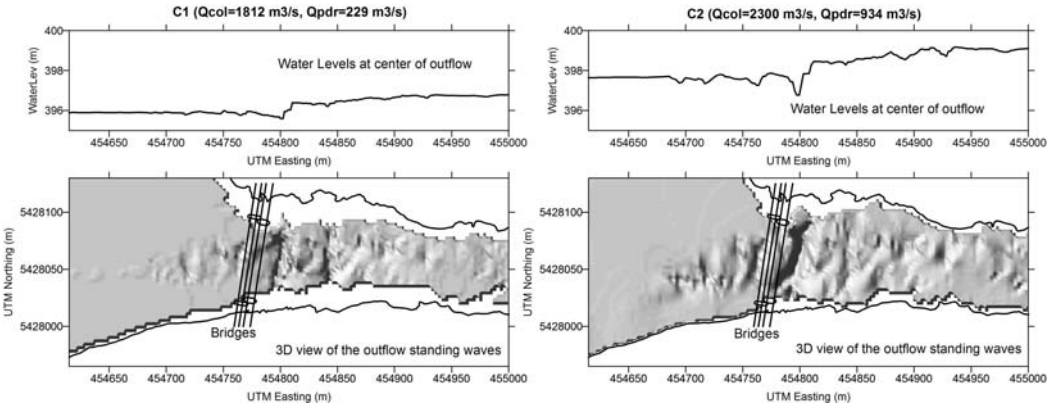

Figure 4. Model results of standing waves in outflow area.

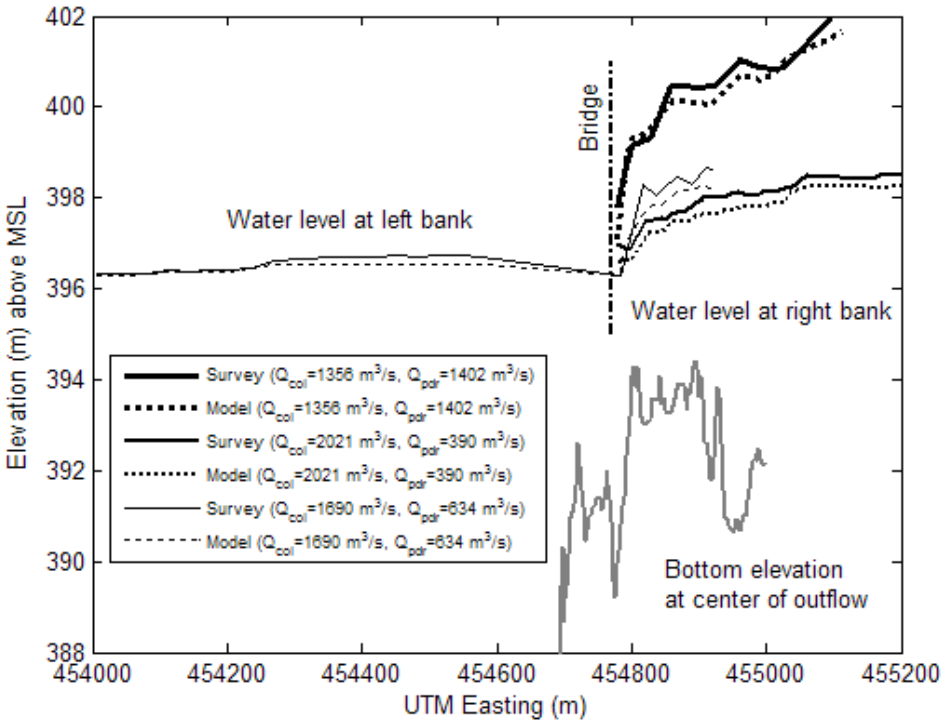

Figure 5. Modeled and observed water surface elevations in the outflow area.

Temperatures

Field measurements (Birch, 1994; Hildebrand and Fissel, 1997) revealed considerable spatial variations of temperature in the confluence area, depending on the temperatures of the water discharged from each river, which is highly seasonal in nature and on the flow conditions. In summer, the water temperature of the Pend d'Oreille River flow is 2 to 4 ºC higher than that of the upstream Columbia River, while in winte, it is 1 to 3 ºC lower than the upstream Columbia River. Hildebrand (2001) has shown that water temperatures represent another important environmental parameter for fish habitat at the confluence of the Columbia and Pend d'Oreille rivers. For example, water temperatures in the spawning period of late spring to mid-summer, if too high, can be detrimental to egg survival in the white sturgeon spawning and egg deposition area. On the other hand, the temperature-induced water density gradient may have considerable effect on flow patterns in the confluence area (Fissel and Jiang, 2002; Jiang and Fissel, 2002). Therefore, water temperature simulations and baroclinic effects were directly applied to the calibration and validation cases with water temperature measurements (Table 1). A standard equation of state of seawater described in Millero and Poisson (1980) was used to calculate water density with salinity set to zero. As shown in Figure 6, modeled water temperatures are in generally good agreement with observations in terms of thermal plume patterns in the confluence area.

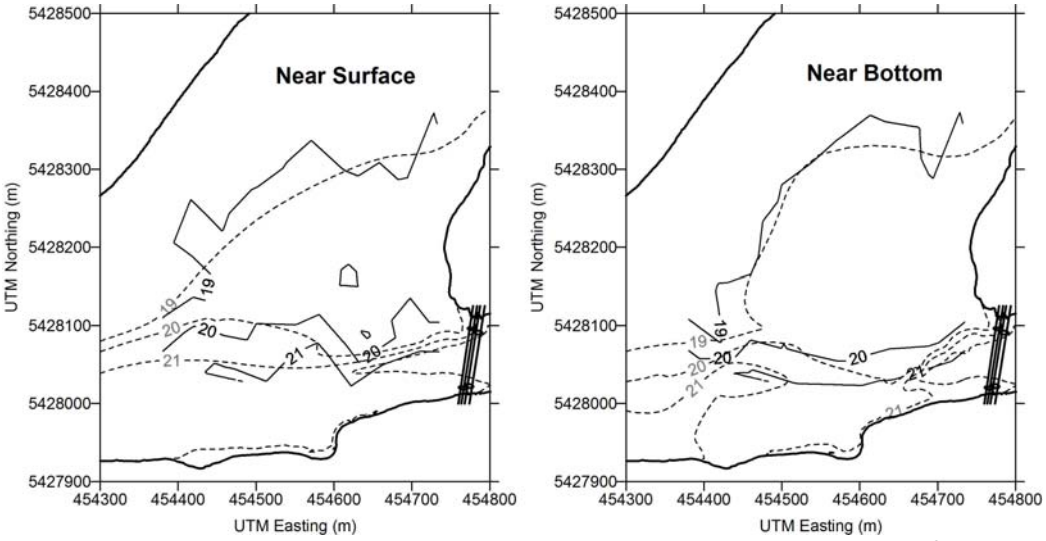

Figure 6. Modeled (dashed contours) and observed (solid contours) results of near-surface and near-bottom water temperatures in the confluence area for case V1.

Validation Statistics

The statistical framework involves a set of observed flow vectors, $\boldsymbol{V}$, the corresponding model flow vectors, $\boldsymbol{M}$, and weighting functions, w: $\{(\boldsymbol{V}_i, \boldsymbol{M}_i, w_i), i = 1, 2, \ldots\ldots, n\}$. The weighting function, w (≤ 1), is included to represent equal effects per unit area represented by each observational flow sample, which has the form as follows.

$$w = \max\left\{\left(\frac{\bar{r}}{\bar{r}_{\max}}\right)^2, \left(1-\frac{N}{N_{\max}}\right)\right\}, \quad \text{with } 0 < w <= 1 \tag{2}$$

where N is the number of available observations near a sampling point within an area with a radius r <= 25 m (data sampling spacing), $\bar{r}$ denotes the mean distance between the observation sites and current sampling point, and $\bar{r}_{\max}$ and $N_{\max}$ are respectively the maximum values of $\bar{r}$ and N in the entire domain.

The following key validation statistics are computed (Murphy and Winkler, 1987).

- Vector mean of observation and model, $\overline{\boldsymbol{V}}$ and $\overline{\boldsymbol{M}}$

$$\overline{\boldsymbol{V}} = \frac{\sum_{i=1}^{n} w_i \boldsymbol{V}_i}{\sum_{i=1}^{n} w_i}, \qquad \overline{\boldsymbol{M}} = \frac{\sum_{i=1}^{n} w_i \boldsymbol{M}_i}{\sum_{i=1}^{n} w_i} \tag{3}$$

- Bias velocity, *bias*

$$bias = \frac{\sum_{i=1}^{n} w_i (\boldsymbol{M}_i - \boldsymbol{V}_i)}{\sum_{i=1}^{n} w_i} \tag{4}$$

- Root Mean Square Difference, *RMSD*

$$RMSD = \sqrt{\frac{\sum_{i=1}^{n} w_i (\boldsymbol{M}_i - \boldsymbol{V}_i)^2}{\sum_{i=1}^{n} w_i}} \tag{5}$$

- Correlation coefficient, *R*, computed as least square fit of model to observation

$$R = \frac{\sum_{i=1}^{n} w_i \left(\boldsymbol{M}_i - \overline{\boldsymbol{M}}\right)\left(\boldsymbol{V}_i - \overline{\boldsymbol{V}}\right)}{\sqrt{\sum_{i=1}^{n} w_i \left(\boldsymbol{M}_i - \overline{\boldsymbol{M}}\right)^2 \sum_{i=1}^{n} w_i \left(\boldsymbol{V}_i - \overline{\boldsymbol{V}}\right)^2}} \tag{6}$$

Table 2 shows the vertically average statistical properties for model calibration and validation. Validation cases V3, V4 and V6 are not included because the current measurements for V3, V4, and V6 are limited in the Pend d'Oreille River. The model performance attained for C1, C2, V1, V2, V5, and V7, as listed in Table 2, is good as evidenced by the high correlation scores, mostly greater than 0.8, and low bias velocities, mostly less than 0.03 m/s. The correlation coefficients for cases V2 and V5 are slightly lower, ranging from 0.71 to 0.78, which is believed to be mainly caused by limited number of observations, especially at near-bottom levels (only 10 to 22). These two cases deal with low Columbia River discharges combined with low Pend d'Oreille discharges (Table 1). The limited number of observations also makes the statistical results more prone to the effects by one or two uncertain data measured. For all six cases compared in Table 2, the RMSD values range from 0.20 to 0.36 m/s. From detailed analysis of the differences between model results and observations, it is

found that significant contributions to the RMSD values usually come from several large differences located in the small sub-area of the strong shear zone between the Waneta Eddy and Pend d'Oreille outflow, where most notable differences between modeled and observed flows occur (Fissel and Jiang, 2002).

Table 2. Vertically-averaged statistical properties.

Model case	*Bias* (m/s)	*RMSD* (m/s)	*R*		
			East	North	Average
C1	0.01	0.28	0.81	0.79	0.80
C2	0.03	0.33	0.92	0.86	0.89
V1	0.03	0.34	0.84	0.86	0.85
V2	0.01	0.20	0.78	0.71	0.75
V5	0.04	0.25	0.77	0.75	0.76
V7	0.02	0.36	0.87	0.87	0.87

Summary and Conclusion

The confluence area of the Columbia and Pend d'Oreille rivers has some significant morphological and circulation features, which are known as important habitats for white sturgeon, such as deep water, low-speed Waneta Eddy for sturgeon rearing and feeding and the jet-like high-speed Pend d"Oreille River outflow for sturgeon spawning. To predict the potential impact of the proposed WEP project on the circulation patterns in this area, the high resolution, 3D coastal circulation numerical model COCIRM was successfully adapted and optimized through extensive calibration and validation processes. It is shown that the model provides very good performance and is capable of accurately reproducing and predicting the complex flow dynamics at the confluence. On the basis of a favourable anonymous peer review of the model by a numerical modeller of the Department of Fisheries and Oceans, Canada, this 3D numerical model was accepted as an acceptable basis for the WEP environmental assessment purposes by the Canadian regulatory agencies and First Nation groups in 2003. Based in part on these numerical model studies, environmental approval for the Waneta Expansion Project was granted in 2007.

The detailed model calibration and validation results revealed that the dynamic processes of circulation patterns in this confluence area occur due to the significant morphological features and driving forces of the two major rivers. The most important morphological features include the gravel bar, embayment of the Waneta Eddy area with deep water depth, and the shallow and narrow passageway of the Pend d'Oreille River, which causes the supercritical condition of the outflow and the associated large standing waves. Overall, the modeled circulation patterns for different flow conditions are in good agreement with the available boat-based ADCP data. The modeled standing wave features for all calibration and validation cases are in reasonable agreement with field visual and photographic observations. The outflow standing waves are highly three dimensional with a rough water surface at the center and relatively smooth water surface near the shore. The comparisons between model and measured water surface elevations at the river bank of the outflow show that the model can accurately reproduce the large water drop (1 to 4 m) under the bridges as well as the water surface profiles both upstream and downstream of this water drop.

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